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Erratum to "Single machine scheduling problems under the effects of nonlinear deterioration and time-dependent learning" [Math. Comput. Modelling 50 (2009) 401–406]

1. Introduction

Recently, Toksar et al. [1] studied several single machine scheduling problems under the joint effects of nonlinear job deterioration and time-dependent learning. The time-dependent learning effect of a job was first proposed by Kuo and Yang [2]. Kuo and Yang [2] introduced a model consisting of a time-dependent learning effect (rather than a position-dependent learning effect). Under their model, the actual processing time of a job is a function of the total normal processing time of jobs scheduled in front of it, i.e.,

$$p_{jr}^A = (1 + p_{[1]} + p_{[2]} + \cdots + p_{[r-1]})^a p_j = \left(1 + \sum_{k=1}^{r-1} p_{[k]}\right)^a p_j \tag{1}$$

where p_j is the normal processing time of the job J_j, $p_{[r]}$ is the normal processing time of the job scheduled in position r, p_{jr}^A is the actual processing time of the job J_j and $a \leq 0$ is the learning index. Toksar et al. [1] use this type of learning effect in their model.

Toksar et al. [1] also considered the nonlinear deterioration effect proposed by Alidaee and Womer [3], i.e.,

$$p_{jr}^A = p_j + \alpha t_r^b \tag{2}$$

where t_r^b is the starting time of the job J_j, and α ($\alpha > 0$) and b ($b \geq 0$) are parameters of the nonlinear deterioration effect, which determine the increase in the processing time of a job per unit delay in its starting time.

The proposed effects of nonlinear deterioration and the time-dependent learning in the model are Toksar et al. [1] are described as follows.

A set of n jobs is available for processing on a single machine at time zero. If the job J_j, $j = 1, 2, \ldots, n$, is scheduled in position r in a sequence, its actual processing time is

$$p_{jr}^A = [p_j + (\alpha \times t_r^b)] \left(1 + \sum_{k=1}^{r-1} p_{[k]}\right)^a. \tag{3}$$

The objectives considered are the makespan $C_{\max}$, the sum of completion times $\sum C_j$, the sum of completion times squared $\sum C_j^2$ and the maximum lateness $L_{\max}$ of a given permutation.

Toksar et al. [1] gave the following results for the single-machine scheduling problems.

Theorem 1. *The makespan problem on a single machine under the effects of nonlinear deterioration and time-dependent learning,* $1|p_{jr}^A = [p_j + (\alpha \times t_r^b)] \left(1 + \sum_{k=1}^{r-1} p_{[k]}\right)^a |C_{\max}$, *can be solved optimally by sequencing jobs in non-decreasing order of their basic processing times (SPT rule).*

Theorem 2. *The total completion time problem on a single machine under the effects of nonlinear deterioration and time-dependent learning,* $1|p_{jr}^A = [p_j + (\alpha \times t_r^b)] \left(1 + \sum_{k=1}^{r-1} p_{[k]}\right)^a | \sum C_j$, *can be solved optimally by sequencing jobs in non-decreasing order of their basic processing times (the SPT rule).*

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Theorem 3. *The total completion time (square) problem on a single machine under the effects of nonlinear deterioration and time-dependent learning,* $1|p_{jr}^A = [p_j + (\alpha \times t_r^b)]\left(1 + \sum_{k=1}^{r-1} p_{[k]}\right)^a | \sum C_j^2$*, can be solved optimally by sequencing jobs in non-decreasing order of their basic processing times (the SPT rule).*

Theorem 4. *The maximum lateness problem on a single machine under the effects of nonlinear deterioration and time-dependent learning,* $1|p_{jr}^A = [p_j + (\alpha \times t_r^b)]\left(1 + \sum_{k=1}^{r-1} p_{[k]}\right)^a |L_{\max}$*, if jobs have agreeable due dates (i.e.* $p_u < p_v$ *implies due dates* $d_u < d_v$ *for all jobs* J_u *and* J_v*) can be solved optimally by sequencing the jobs in non-decreasing order of their due dates* d_j *(the EDD rule).*

2. A counter-example

In the following example, we show that the results of Theorems 1–4 are not correct by giving a counter-example.

Counter-example 1. Let $n = 3$; $p_1 = 10$; $p_2 = 15$; $p_3 = 16$; $d_1 = d_2 = d_3 = 10$; $\alpha = 100$; $a = -4$; $b = 1$. If the jobs are scheduled to be processed according to the SPT rule, and the sequence of the jobs is J_1, J_2 and J_3, then according to the result of Theorem 1,

$$\begin{aligned}
&p_{[1]} = 10\\
&p_{[2]} = [p_2 + (\alpha \times t_2^b)](1 + p_{[1]})^a = (15 + 100 \times 10)(1 + 10)^{-4} = 0.0693\\
&p_{[3]} = [p_3 + (\alpha \times t_3^b)](1 + p_{[1]} + p_{[2]})^a = [16 + (100 \times 10.0693)](1 + 10 + 15)^{-4} = 0.0022\\
&C_1 = p_{[1]} = 10\\
&C_2 = p_{[1]} + p_{[2]} = 10 + 0.0693 = 10.0693\\
&C_3 = p_{[1]} + p_{[2]} + p_{[3]} = 10 + 0.0693 + 0.0022 = 10.0715.
\end{aligned}$$

Then, the makespan $C_{\max}$ and the total completion time $\sum C_j$ are calculated as follows:

$$\begin{aligned}
C_{\max} &= C_3 = p_{[1]} + p_{[2]} + p_{[3]}\\
&= 10 + (15 + 100 \times 10)(1 + 10)^{-4} + [16 + (100 \times 10.0693)](1 + 10 + 15)^{-4}\\
&= 10.0715
\end{aligned}$$

$$\sum C_j = C_1 + C_2 + C_3 = 10 + 10.0693 + 10.0715 = 30.1408$$
$$\sum C_j^2 = C_1^2 + C_2^2 + C_3^2 = 10^2 + 10.0693^2 + 10.0715^2 = 302.8259.$$

However, if the sequence of the jobs is J_1, J_3 and J_2, then we have

$$\begin{aligned}
&p_{[1]} = 10\\
&p_{[2]} = [p_2 + (\alpha \times t_2^b)](1 + p_{[1]})^a = (16 + 100 \times 10)(1 + 10)^{-4} = 0.0694\\
&p_{[3]} = [p_3 + (\alpha \times t_3^b)](1 + p_{[1]} + p_{[2]})^a = [15 + (100 \times 10.0694)](1 + 10 + 16)^{-4} = 0.0019\\
&C_1 = p_{[1]} = 10\\
&C_2 = p_{[1]} + p_{[2]} = 10 + 0.0694 = 10.0694\\
&C_3 = p_{[1]} + p_{[2]} + p_{[3]} = 10 + 0.0694 + 0.0019 = 10.0713
\end{aligned}$$

$$\begin{aligned}
C_{\max} &= C_3 = p_{[1]} + p_{[2]} + p_{[3]}\\
&= 10 + (16 + 100 \times 10)(1 + 10)^{-4} + [15 + (100 \times 10.0694)](1 + 10 + 16)^{-4}\\
&= 10.0713
\end{aligned}$$

$$\sum C_j = C_1 + C_2 + C_3 = 10 + 10.0694 + 10.0713 = 30.1407$$
$$\sum C_j^2 = C_1^2 + C_2^2 + C_3^2 = 10^2 + 10.0694^2 + 10.0713^2 = 302.8239.$$

Obviously, the SPT sequence is not the optimal schedule for the problems, $1|[p_r + (\alpha \times t_r^b)]\left(1 + \sum_{k=1}^{r-1} p_{[k]}\right)^a |C_{\max}$, $1|[p_r + (\alpha \times t_r^b)]\left(1 + \sum_{k=1}^{r-1} p_{[k]}\right)^a | \sum C_j$ and $[p_r + (\alpha \times t_r^b)]\left(1 + \sum_{k=1}^{r-1} p_{[k]}\right)^a$. Hence, the results of Theorems 1–3 are incorrect. As for the problem $1|[p_r + (\alpha \times t_r^b)]\left(1 + \sum_{k=1}^{r-1} p_{[k]}\right)^a |C_{\max}$, when $d_1 = d_2 = d_3 = 10$, finding the optimal sequence of the problem $1|[p_r + (\alpha \times t_r^b)]\left(1 + \sum_{k=1}^{r-1} p_{[k]}\right)^a |L_{\max}$ is equivalent to finding that of the problem $1|[p_r + (\alpha \times t_r^b)]\left(1 + \sum_{k=1}^{r-1} p_{[k]}\right)^a |C_{\max}$; hence, Theorem 4 in [1] is also incorrect.

3. Conclusion

Although the results of the paper are not correct, the authors considered several single machine problems under the simultaneous effects of nonlinear deterioration and time-dependent learning. This is the most general model studied to date. It is an attempt to develop a framework for better describing real life systems where the rate of deterioration increases or decreases over time and where the learning is driven by time rather than by the number of completed tasks. Such a scenario can arise in many realistic situations.

Hence, it is worthwhile to discuss the complexity of scheduling problems with the proposed model and to find sufficient conditions to ensure that the theorems of the paper remain correct.

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